# Algebra 2, Quarter 1, Unit 1.1
## Linear Programming

### Overview

**Number of Instructional Days:** 8  
(1 day = 45 minutes)

**Content to Be Learned**
- Interpret the verbal model to define the variables and write the objective function.
- Represent constraints as equations or inequalities.
- Graph systems of equations and/or inequalities on coordinate axes with labels and scales and determine a feasible region.
- Identify important quantities in a practical situation and map their relationships.
- Interpret the corner points to find the optimal solution.
- Identify and interpret solutions as viable or non-viable options in a real-world context.

**Mathematical Practices to Be Integrated**
- Make sense of problems and persevere in solving them.
- Analyze information and goal.
- Create a linear programming model that appropriately represents the situation described.
- Explain correspondence between the verbal description and the graph.
- Model with mathematics.
- Create a linear programming model and represent it graphically to identify the feasible region.
- Analyze linear relationships mathematically to draw conclusions.
- Identify important quantities in a practical situation and map their relationships.
- Interpret the corner points to find the optimal solution.
- Use appropriate tools strategically.
- Analyze graphs of systems of linear equations and inequalities to determine feasible regions, and analyze corner points to determine the solution of a linear program problem using appropriate tools.

### Essential Questions

- What process would you use to optimize the objective function?
- What is the feasible region, and how does it contribute to identifying solutions to a problem?
- How might the number of constraints affect possible solutions?
- How would you apply what you have learned in systems of inequalities to linear programming?
- What are some real-world situations that can be solved using linear programming?
Written Curriculum

Common Core State Standards for Mathematical Content

Creating Equations*  A-CED

Create equations that describe numbers or relationships [Equations using all available types of expressions, including simple root functions]

A-CED.2  Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

A-CED.3  Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Common Core Standards for Mathematical Practice

1  Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4  Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of
the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grades 7, 8, and 9, students graphed linear equations and inequalities in two variables, solved 2 x 2 linear systems, graphed solutions to a system of inequalities in two variables, created a mathematical model from a verbal description, interpreted solutions, and determined appropriate domains.

Current Learning

Students create a mathematical model from a verbal description. They create equations and inequalities in one or two variables to represent relationships between quantities. They represent constraints as equations or inequalities. They graph systems of equations and/or inequalities on coordinate axes with labels and scales, and interpret solutions as viable or non-viable options in a modeling context.

Future Learning

Students will access prior knowledge when they determine the domain that reflects the context of a situation and when they determine optimal solutions. They will graph and interpret functions with more than one variable, identify bounded and unbounded regions, and determine when a problem has a unique solution, no solution, or infinitely many solutions. This knowledge transfers to calculus and college-level business courses.
Additional Findings

*Beyond Numeracy* describes how linear programming is a method for maximizing or minimizing some quantity, while it is a useful technique to improve performance in businesses (pp. 133–135).

“Linear programming (LP) is one of the most widely applied O.R. techniques …”
www.economicsnetwork.ac.uk/cheer/ch9_3/ch9_3p07.htm

“The Traveling Salesman Problem is one of the most intensively studied problems in computational mathematics.” www.tsp.gatech.edu/

Teacher Notes:

Students often have difficulty distinguishing between constraints and the objective function. Some teachers have found it effective to focus on the objective function first and have students check to make sure that function “answers the question asked.”
Algebra 2, Quarter 1, Unit 1.2
Complex Numbers

Overview

Number of Instructional Days: 6  (1 day = 45 minutes)

Content to Be Learned

- Use the definition $i^2 = -1$ to simplify radicals.
- Use the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- Solve quadratic equations with real coefficients that have real and complex solutions in the form of $a \pm bi$.
- Use the process of factoring and completing the square in quadratic functions to show real and complex zeros.

Mathematical Practices to Be Integrated

Attend to precision.

- Use appropriate definitions and terminology for complex, real, pure imaginary and imaginary.
- Look for and make use of structure.
- Demonstrate an understanding of the structure of complex numbers.

Essential Questions

- Why do imaginary numbers exist?
- How do you add, subtract, and multiply complex numbers?
- When does a quadratic equation have imaginary solutions?
- How do you find imaginary solutions for quadratic equations?
**Written Curriculum**

**Common Core State Standards for Mathematical Content**

**The Complex Number System**

<table>
<thead>
<tr>
<th>N-CN</th>
<th>Perform arithmetic operations with complex numbers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-CN.1</td>
<td>Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.</td>
</tr>
<tr>
<td>N-CN.2</td>
<td>Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</td>
</tr>
</tbody>
</table>

**Use complex in polynomial identities and equations.**

| N-CN.7 | Solve quadratic equations with real coefficients that have complex solutions. |

**Reasoning with Equations and Inequalities**

<table>
<thead>
<tr>
<th>A-REI</th>
<th>Solve equations and inequalities in one variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-REI.4</td>
<td>Solve quadratic equations in one variable.</td>
</tr>
</tbody>
</table>

| b. | Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$. |

**Common Core Standards for Mathematical Practice**

6 **Attend to precision**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see

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complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

**Clarifying the Standards**

**Prior Learning**

In prior courses, students have operated with rational number systems and have understood the basic concepts of functions including linear and quadratic. In eighth grade, students learned that there are numbers that are not rational, and they approximated them by using rational numbers.

**Current Learning**

Building on A-REI.4a and A-REI.4b, students are introduced to the relation $i^2 = -1$ and to the complex number system. Students solve quadratic equations and inequalities with complex solutions, and they perform operations with complex numbers.

**Future Learning**

Students will interpret and model quadratic relationships between two quantities. They will use factoring, completing the square, and graphing to identify zeros, intercepts, and intervals where the functions are increasing and decreasing and positive and negative. Students will also find extreme values of functions. A key feature in graphing will be recognizing that not all zeros of functions are $x$-intercepts. In the fourth-year course, students will apply the concepts of complex roots to higher-degree polynomial functions, and they will apply the Fundamental Theorem of Algebra. Students will use conjugates to find quotients of complex numbers and represent complex numbers and their operations geometrically on the complex plane.

**Additional Findings**

*Beyond Numeracy* discusses that, given the expanded number system, we can prove the Fundamental Theorem of Algebra. With complex numbers, we soon extended the definition of trigonometric and exponential functions to the domain of complex numbers and generalized analysis to calculus and differential equations and related fields. These technical advances, including the geometrical interpretation of various operations on complex numbers, paved the way for their indispensable use in electrical theory and other physical sciences (pp. 116–117). *Beyond Numeracy* states that quadratics have real-world applications in engineering, physics, and elsewhere. It is more than just plugging in numbers (p. 198).

*Atlas of Science Literacy, Volume 1* indicates that students have difficulty translating graphical and algebraic representations, especially moving from a graph to an equation (p. 114).

*On Core Mathematics Algebra 2* states, “If students have difficulty evaluating $b^2 - 4ac$, you can provide a connection to Mathematical Practice Standard 6 (Attend to precision)” (p. 23).
**Teacher Notes**

Teachers may consider including rational expressions with complex numbers and rationalization of denominator.

Teachers must include the following:

- multiplication of radical binomials
- multiplication of complex binomials
- rational expressions with complex numbers
- rationalization of binomial radical denominators
- rationalization of denominator with complex number (optional)
Algebra 2, Quarter 1, Unit 1.3
Functions Overview

Overview

Number of Instructional Days: 6 (1 day = 45 minutes)

Content to Be Learned

- Graph and identify key features of the parent functions \( f(x) = x^2, f(x) = |x|, f(x) = \sqrt{x}, \)
  \( f(x) = x^3, f(x) = \sqrt[3]{x}, f(x) = \frac{1}{x}. \)
- Explore transformations of selected functions.
- Graph piece-wise and step functions.
- Given a graph, determine domain, range, intercepts, end behavior, minimums, maximums, symmetries, and intervals where the function is positive, negative, increasing, decreasing, and/or constant.
- Find inverses of selected functions: simple quadratic, radical, and rational.

Mathematical Practices to Be Integrated

- Attend to precision.
- Understand function notation.
- Communicate precisely to others.
- Examine each parent function and make explicit use of each function type.
- Look for and make use of structure.
- Discern pattern or structure within functions.
- Determine the similarities and differences of each parent function.
- Recognize patterns of end behaviors.
- How would you compare and contrast the domain, range, and end behavior of the following parent functions and transformations of them: \( f(x) = x^3, f(x) = |x|, f(x) = \sqrt{x}, f(x) = \frac{1}{x}, \sqrt[3]{x}, \) piecewise, and step functions?
- How would you determine minimums, maximums, and intercepts of each parent function?
- How would you determine the intervals in which a function is increasing, decreasing, and/or constant?

Essential Questions

- What are the various representations of each of the following parent functions: \( f(x) = x^2, f(x) = |x|, f(x) = \sqrt{x}, f(x) = x^3, f(x) = \sqrt[3]{x}. \) ?
- How do transformations of functions compare to the parent graph?
- What is the connection among the various representations of piecewise and step functions?
Written Curriculum

Common Core State Standards for Mathematical Content

Building Functions

| F-BF.3 | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. |

Build new functions from existing functions

| F-BF.4 | Find inverse functions. |
| a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. |

Interpreting Functions

| F-IF.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| F-IF.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. |

Analyze functions using different representations

| F-IF.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [Note: Focus on using key features to guide selection of appropriate type of model function.] |
Common Core Standards for Mathematical Practice

6  Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7  Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

Clarifying the Standards

Prior Learning

In grade 8, students interpreted and constructed linear functions.

In algebra 1, students learned the concepts of a function and the use of function notation. They have interpreted linear, exponential, and quadratic functions in applications or in terms of a context. They have analyzed linear, exponential, quadratic, absolute value, step, and piecewise functions using a graphical representation.

Current Learning

Students graph and identify key features of the parent functions $f(x) = x^2$, $f(x) = |x|$, $f(x) = \sqrt{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$. They also graph piece-wise and step functions.

Future Learning

In the fourth-year course, students will analyze logarithmic and trigonometric functions using different representations.

Additional Findings

Benchmarks for Science Literacy states, “In modeling phenomena, students should encounter a variety of common kinds of relationships depicted in graphs…” (p. 220).

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In *A Research Companion to Principles and Standards for School Math*, Slavit states, “Students [have] difficulties with domain, range, and distinguishing from functions versus non-functions.” He suggests that “focusing on a covariational approach for deeper understanding of functions” is necessary. (p. 142)

**Teacher Notes:**

You may want to consider …

- adding operations on functions (including addition, subtraction, multiplication, division, and composition) and associated domains.
- addressing the determination of whether a function is even or odd.
- discussing the determination of inverse functions.
Algebra 2, Quarter 1, Unit 1.4
Quadratic Functions and Equations

Overview

Number of Instructional Days: 20  (1 day = 45 minutes)

Content to Be Learned

- Create equations and inequalities in one variable and use them to solve quadratic problems.
- Create and graph equations and inequalities in two variables to represent quadratic relationships between quantities.
- Interpret key features and sketch graphs of quadratic relationships from verbal descriptions including zeros, intercepts, intervals of increase and decrease, intervals of positive and negative values, extreme values, symmetries, and end behaviors.
- Use the domain to determine the reasonableness of solutions to quadratic applications.
- Compare the properties of two quadratic functions, each represented in a different way (i.e., one in algebraic form and one in table form).

Mathematical Practices to Be Integrated

- Make sense of problems and persevere in solving them.
- Think about simpler problems to help solve quadratic problems with complex solutions.
- Work between different algebraic and graphical representations with quadratic functions.
- Model with mathematics.
- Apply knowledge of quadratic functions to problems with real-world applications (e.g., create a revenue function from a survey. See writing team notes for reference.)
- Analyze quadratic relationships to draw conclusions.
- Look for and make use of structure.
- Understand that using a table or graph may be helpful in understanding the symmetry of a parabola and how symmetry can be used to locate points on the graph of a quadratic function.
- How do you determine where the graph of a quadratic function crosses the x-axis?
- How are quadratic models used to solve real-world problems?

Essential Questions

- How does the graph of \( g(x) = (x - h)^2 + k \) compare with the graph of \( f(x) = x^2 \)?
- How does the graph of \( g(x) = ax^2 \) differ from the graph of \( f(x) = x^2 \)?
- How can you graph the function \( f(x) = a(x - h)^2 + k \)?
- How do you convert quadratic functions to the vertex form \( f(x) = a(x - h)^2 + k \)?
Common Core State Standards for Mathematical Content

The Complex Number System

N-CN

N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.

Creating Equations*

A-CED

Create equations that describe numbers or relationships [Equations using all available types of expressions, including simple root functions]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

Building Functions

F-BF

Build a function that models a relationship between two quantities [For F-BF.1, 2, linear, exponential, and quadratic]

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Interpreting Functions

F-IF

Interpret functions that arise in applications in terms of the context [Emphasize selection of appropriate models]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.*
Analyze functions using different representations [Focus on using key features to guide selection of appropriate type of model function]

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

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4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

Clarifying the Standards

Prior Learning

In algebra 1, students used the structure of an expression to factor the difference of squares (A-SSE.2), and they factored quadratic expressions, found function zeros, completed the square, and found function maximums and minimums (A-SSE.3a and A-SSE.3b). Students completed the square to derive the quadratic formula and to transform quadratic equations. They solved quadratic equations by inspection, taking square roots, completing the square, using the quadratic formula, and factoring. Students learned to recognize complex solutions as those for which the solution is not real (A-REI.4a and A-REI.4b). Previously in algebra 2, students were introduced to the definition $i^2 = -1$ and the complex number system, and they solved quadratic equations and inequalities with complex solutions; students also performed operations with complex numbers.

Current Learning

Students interpret and model quadratic relationships between two quantities. They use factoring, completing the square, and graphing to identify zeros, intercepts, and intervals where the functions are increasing and decreasing and positive and negative, and they find extreme values. A key feature in graphing is recognizing that not all zeros of functions are $x$-intercepts.

Future Learning

In the fourth-year course, students will apply the concepts of complex roots to higher-degree polynomial functions and apply the Fundamental Theorem of Algebra. The will use conjugates to find quotients of complex numbers and represent complex numbers and their operations geometrically on the complex plane.
**Additional Findings**

*Beyond Numeracy* discusses that, given the expanded number system, we can prove the Fundamental Theorem of Algebra. With complex numbers, we soon extended the definition of trigonometric and exponential functions to the domain of complex numbers and generalized analysis to calculus and differential equations and related fields. These technical advances, including the geometrical interpretation of various operations on complex numbers, paved the way for their indispensable use in electrical theory and other physical sciences (pp. 116–117). *Beyond Numeracy* states that quadratics have real-world applications in engineering, physics, and elsewhere. It is more than just plugging in numbers (p. 198).

*Atlas of Science Literacy, Volume 1* indicates that students have difficulty translating graphical and algebraic representations, especially moving from a graph to an equation (p. 114).

**Teacher Notes:**

This unit of study should be restricted to quadratics that can be written in the form $ax^2+bx+c$. 