Algebra 1, Quarter 1, Unit 1.1

Creating and Interpreting Expressions and Equations

Overview

Number of Instructional Days: 10 (1 day = 45 minutes)

Content to Be Learned

(For each of the bullets below, the expressions and equations refer to, but are not limited to, linear, quadratic, and exponential relationships.)

- Simplify expressions.
- Write expressions in equivalent forms.
- Identify parts of expressions and equations and view one or more of their parts as a single entity.
- Interpret the parts of expressions and equations related to real-world situations.
- Use tables, graphs, and equations to represent verbal descriptions.
- Model relationships by creating expressions and equations.
- Recognize and use patterns to write expressions and equations.
- Create equations in two or more variables.
- Use tables to graph equations on coordinate axes.
- Identify independent and dependent variables.

Mathematical Practices to Be Integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of expressions and equations to themselves.
- Determine if the expression or equation makes sense in the context of the situation.
- Explain correspondences between equations, verbal descriptions, tables, and graphs.

Look for and make use of structure.

- Draw diagrams of important features and relationships; graph data; and search for regularity, trends, and patterns.
- Write algebraic expressions and equations as single objects or as being composed of several objects and use them to represent complicated ideas.

Model with mathematics.

- Identify important quantities in a practical situation and map their relationships using such tools as diagrams, tables, graphs, and formulas.
- Interpret mathematical results in the context of the situation.
- Reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Essential Questions

- How can you use terms, factors, and coefficients to simplify expressions?
- Why are variables used in algebraic expressions and equations?
- What is the difference between an algebraic expression and an algebraic equation?
- How can tables, graphs, and equations be used to represent verbal descriptions and vice-versa?
- How can you use expressions and equations to model real-world situations?
- How can you use expressions and equations/formulas to solve problems?

Written Curriculum

Common Core State Standards for Mathematical Content

Seeing Structure in Expressions **A-SSE**

Interpret the structure of expressions *[Such as, but not limited to linear, exponential, quadratic expressions]*

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

Creating Equations★ **A-CED**

Create equations that describe numbers or relationships [*Such as, but not limited to linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only]*

A-CED.1 Create equations ~~and inequalities~~ in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 6, students wrote, read, and evaluated expressions in which letters stood for numbers. They identified parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient). They viewed one or more parts of an expression as a single entity. They evaluated expressions as specific values of their variables. They applied properties of operations to generate equivalent expressions and identified when two expressions were equivalent. (6.EE.2, 3, 4)

By grade 7, the students began to recognize that rewriting expressions in different forms could be useful in problem solving. (7.EE.2)

In grade 8, students graphed proportional relationships, interpreting the unit rate as the slope of the graph. Also, they interpreted the equation $y = mx + b$ as defining a linear function whose graph is a line.

Current Learning

Students interpret the parts of an expression and view one or more of its parts as a single entity. They write expressions to represent real-world quantities, and they recognize the real-world significance of the parts of the expression. Students extend their knowledge of properties of operations to rewrite expressions, gain fluency, and engage in the reasoning of expressions. Students recognize patterns in tables and graphs and write expressions and equations that represent these patterns.

Future Learning

Students will use the knowledge gained in this unit throughout the rest of algebra 1, and they will continue to expand their use of expressions throughout the rest of their high school years and into the future. More immediately, students will use this knowledge in unit of study 1.2 of this course as they solve one-variable equations and inequalities.

Additional Findings

Fluency with algebraic symbolism helps students represent and solve problems in many areas of the curriculum. According to *Principles and Standards of School Mathematics*, "Students should be able to operate fluently on algebraic expressions, combining them and re-expressing them in alternative forms."

Algebra 1, Quarter 1, Unit 1.2

Solving Linear Equations and Inequalities in One Variable

Overview

Number of Instructional Days: 15 (1 day = 45 minutes)

Content to Be Learned

- Create single-variable linear equations and inequalities, including compound inequalities, and use them to solve problems.
- Solve and justify the steps involved in solving single-variable equations and inequalities, including those with coefficients represented by letters.
- Identify and use appropriate units of measure to solve single-variable equations and inequalities.
- Solve equations and inequalities, including formulas, for any variable.

Mathematical Practices to Be Integrated

Make sense of problems and persevere in solving them.

- Plan a solution pathway rather than jumping into a solution attempt.
- Check answers and ask, "Does my solution make sense?"

Reason abstractly and quantitatively.

- Represent a situation symbolically.
- Manipulate equations and inequalities in order to obtain a solution.

Model with mathematics.

- Apply the mathematics known to solve problems arising in everyday life, society, and the workplace.
- Interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Essential Questions

- What are the similarities and differences between algebraic equations and inequalities, and what is the significance of the solutions to both?
- How is solving an inequality similar to solving an equation?
- When would it be useful and/or necessary to solve an equation for another variable within that equation?
- How do you represent the solutions to an inequality or to an absolute value equation on a number line?
- Why is it possible to have no solution or infinitely many solutions to an equation or an inequality?
- How do you determine what unit is best to use to achieve a level of accuracy appropriate to what is being measured?

Written Curriculum

Common Core State Standards for Mathematical Content

Quantities★ N-Q

Reason quantitatively and use units to solve problems. [*Foundation for work with expressions, equations, and functions*]

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; ~~choose and interpret the scale and the origin in graphs and data displays.~~★

N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Creating Equations★ A-CED

Create equations that describe numbers or relationships [*Linear~~, quadratic, and exponential (integer inputs only); for A.CED.3 linear only~~*]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear ~~and quadratic~~ functions~~, and simple rational and exponential functions~~*.★

[TEACHER NOTE: *Include compound inequalities and absolute value equations*.]

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*.★

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning [*Master linear; learn as general principle*]

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable [*Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions*]

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 6, students reasoned about and solved one-variable equations and inequalities as a process of answering a question (for example, "Which values make the equation or inequality true?") They also solved equations of the form $x + p = q$ and $xp = q$ for cases in which p, q, and x were non-negative rational numbers. Students also learned that inequalities such as $x > c$ have infinitely many solutions. (6.EE.5, 7, 8)

In grade 7, students continued to solve equations and inequalities, but with positive and negative rational numbers. (7.EE.3, 4)

By grade 8, students solved linear equations and inequalities with rational coefficients, including those equations whose solutions required expanding expressions using the distributive property and combining like terms. They also solved equations with infinitely many solutions and those with no solutions. (8.EE.7)

Current Learning

This unit is both developmental and a reinforcement of students' prior work with solving simple equations. Students build on these skills to solve more complex single-variable equations and inequalities, including linear and quadratic equations, simple rational and exponential equations, compound inequalities, and absolute value equations. Students use and solve literal equations and inequalities related to formulas. They reason quantitatively and use units to solve problems.

Future Learning

Students will use the knowledge gained in this unit throughout the rest of algebra 1, and continue to expand on their equation solving throughout the rest of their high school years, as they solve more complex equations. More immediately, students will use this knowledge in unit of study 1.3 as they graph linear equations and inequalities.

Additional Findings

To solve an equation means to find values of the variable that make the equation true. According to the Common Core State Standards for Mathematics Appendix A, "By the end of 8th grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations of two variables."

Principles and Standards for School Mathematics states, "most algebra students need considerable experience with linear equations before they become comfortable and fluent in solving them. Students' facility with symbol manipulation can be enhanced if it is based on extensive experience with quantities in contexts through which students develop an initial understanding of the meanings and uses of variables and an ability to associate symbolic expressions with problem contexts."

Algebra 1, Quarter 1, Unit 1.3

Linear Equations and Inequalities in Two Variables

Overview

Number of Instructional Days: 15 (1 day = 45 minutes)

Content to Be Learned

- Choose and interpret the scale and the origin in graphs and data displays.
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.
- Graph the solutions to a linear inequality in two variables as a half-plane.
- Choose an appropriate graphing technique based on the form of the equation or inequality (i.e., standard form, slope-intercept form, point-slope form).
- Choose an appropriate graphing technique based on the context of a given situation.
- Determine if a given equation is linear or non-linear.
- Write equations of lines that are parallel or perpendicular to a given line and passing through a given point.

Mathematical Practices to Be Integrated

Make sense of problems and persevere in solving them.

- Determine the best way to graph a line, given the structure of the equation or the context of the question that leads to a linear representation.
- Determine appropriate units when choosing and interpreting the scale and the origin in graphs and data displays.
- Use strategies, such as checking a point, to decide which half-plane should be shaded when graphing inequalities.

Model with mathematics.

- Represent and solve equations and inequalities graphically, including those where students create the equation or inequality from a given situation.
- Construct and compare linear models to non-linear models, such as simple quadratic, simple exponential, absolute value, or simple rational.

Essential Questions

- How can you use linear equations and inequalities to model the results of a real-life situation?
- What are the differences and similarities between the solution set of an inequality versus the solution set of an equation?
- How do you decide the best method to use for graphing an equation or inequality?
- What are the characteristics of an equation that help you to decide whether the graph will be linear or non-linear?
- How do you choose the most appropriate scale on the *x*-axis and the *y*-axis?

Written Curriculum

Common Core State Standards for Mathematical Content

Quantities★ N-Q

Reason quantitatively and use units to solve problems. [*Foundation for work with expressions, equations, and functions*]

N-Q.1 ~~Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas~~; choose and interpret the scale and the origin in graphs and data displays.★

Reasoning with Equations and Inequalities A-REI

Represent and solve equations and inequalities graphically [*Linear ~~and exponential~~; learn as general principle*]

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), ~~and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.~~

Represent and solve equations and inequalities graphically [*Linear ~~and exponential~~; learn as general principle*]

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

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Clarifying the Standards

Prior Learning

In grade 6, students used variables to represent two quantities and identified independent and dependent relationships in graphs, tables, and equations. (6.EE.9)

In grade 7, students analyzed proportional relationships and identified constants of proportionality in tables, graphs, equations, diagrams, and verbal descriptions. (7.RP.2) Students explained what a point on the graph meant in terms of a situation.

In grade 8, students identified equations of the form $y = mx + b$ as linear functions whose graphs are a straight line, and they identified functions that are not linear (e.g., $A = s^2$). (8.F.3) Students defined a function (not using function notation) and created function tables to generate ordered pairs as a means of graphing a function. Students determined rate of change and represented it in multiple ways. (8.F.4) Students graphed proportional relationships and interpreted the unit rate as the slope of the graph. (8.EE.5) They used similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line and derived the equation $y = mx + b$. (8.EE.6)

In units 1.1 and 1.2, students created expressions and equations, and they solved equations in one-variable.

Current Learning

Students deepen their knowledge of graphing and expand their understanding of linear and non-linear equations and their graphs. Students graph equations and inequalities from multiple forms including slope-intercept form, standard form, and point-slope form. They explore the graphs of non-linear equations and inequalities and determine the difference between equations and inequalities that are linear vs. non-linear. Students choose and interpret an appropriate scale and the origin in graphs and data displays.

Future Learning

Students will use the knowledge gained in this unit to investigate and solve systems of linear equations (unit 2.1) and to support their further study of the graphs of more complex functions (unit 2.3). Students will also use the material learned in this unit during unit 3.3 when they fit a linear function to a given set of data. They will continue to build on these concepts in subsequent mathematics courses.

Additional Findings

"In middle grades, students should work more frequently with algebraic symbols than in lower grades. It is essential that they become comfortable in relating symbolic expressions containing variables to verbal, tabular, and graphical representations of numerical and quantitative relations" (*Principles and Standards for School Mathematics*, p. 223).

According to the PARCC Model Content Frameworks for Mathematics, "Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables)."