Precalculus, Quarter 2, Unit 2.1

Introduction to Trigonometric Functions

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Develop even/odd properties of trigonometric functions and use them to simplify expressions and evaluate trigonometric functions.
- Determine the period of each trigonometric function.
- Use special right triangles on the unit circle to evaluate the six trigonometric function values in all four quadrants.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use calculators when differentiating between exact and approximate values.

Attend to precision.

- Communicate accurately when evaluating trigonometric functions.
- Provide exact values and approximate values when appropriate.
- Round answers to the indicated decimal place when appropriate.

Look for and express regularity in repeated reasoning.

- Identify co-terminal angles and periodic nature of trigonometric functions.

Essential questions

- How are reference and co-terminal angles used to evaluate the six trigonometric functions on the unit circle?
- What effect does periodicity and symmetry have on simplifying and evaluating the six trigonometric functions?
- How would even/odd properties of trigonometric functions be used to simplify expressions and evaluate trigonometric functions?

Written Curriculum

Common Core State Standards for Mathematical Content

Trigonometric Functions	**F-TF**

Extend the domain of trigonometric functions using the unit circle

F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

F-TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In geometry, students proved theorems involving similarity, defined trigonometric ratios, and solved problems involving right triangles. (G.SRT.1a, 1b, 2, 3, 6, 7, 8)

In algebra 1, students learned, as general principle, the concept of a function and the use of function notation. (F.IF.1, 2, 3, 5, 8)

In algebra 2, students extended the domain of trigonometric functions using the unit circle, modeled periodic phenomena with the sine and cosine functions, and proved and applied trigonometric identities. (F.T.F. 1, 2, 5, 8)

Students found arc lengths and areas of sectors of circles where radians were introduced as a unit of measure in Geometry. (G.C.5)

In geometry, students applied the Pythagorean Theorem in order to determine the length of a side of a right triangle. (G.GPE.4, 5, 6, 7)

Current Learning

Students calculate the values of the six trigonometric functions using radian measures for all angles using reference angles, co-terminal angles, periodicity, and even/odd properties. They identify symmetry and periodicity of trigonometric functions using the unit circle. Students determine all angles for a given trigonometric value. They use special right triangles in the unit circle to evaluate the six trigonometric functions in all four quadrants.

Future Learning

Later in this course, students will apply these concepts when solving contextual problems involving trigonometric equations, graphing trigonometric functions, and using polar form.

Students will calculate values of trigonometric functions to evaluate definite integrals in calculus.

Additional Findings

According to *Beyond Numeracy* by John Allen Paulos,

- "The sine of an angle varies. It is 0 for an angle of 0 degrees, grows steadily but not linearly to a maximum for 1…" (p. 253)
- "Modern trigonometry is more concerned with periodicity and other properties of the trigonometric functions than it is with their interpretation as ratios." (p. 254)

Teacher Notes:

- Even though only sine, cosine, and tangent functions are mentioned in the standards, it is understood that all six trigonometric functions should be covered.

Other possible content to include:

- Identify reference angles for any given angle on the unit circle.
- Find co-terminal angles for a given angle and generalize for all co-terminal angles.

Precalculus, Quarter 2, Unit 2.2

Graphing Trigonometric Functions

Overview

Number of instructional days: 15 (1 day = 45 minutes)

Content to be learned

- Graph trigonometric functions by hand, identifying zeros, asymptotes, end behavior, period, midline, and amplitude.
- Model periodic phenomena using trigonometric functions.
- Identify inverse trigonometric functions.
- Restrict the domain of trigonometric functions in order to create its inverse.

Mathematical practices to be integrated

Look for and make use of structure.

- Graph trigonometric functions using zeros, extrema, and asymptotes, when applicable.

Look for and express regularity in repeated reasoning.

- Identify the periodic nature of trigonometric functions.
- Identify that trigonometric functions are non-invertible and that domain restrictions are necessary in order to determine inverse trigonometric function values.

Essential questions

- Why is it necessary to restrict the domain of trigonometric functions in order to create their inverse?
- What characteristics of a trigonometric function are necessary in order to graph?
- Given a trigonometric equation of the form $y = A\sin(Bx - C) + D$, or for cosine or tangent, how would changes in the values of *A, B, C,* and *D* influence the graph of the function?
- What real-world scenario can be modeled using a trigonometric function?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions — F-IF

Analyze functions using different representations [*Logarithmic and trigonometric functions*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand ~~in simple cases and using technology for more complicated cases~~.★

d. (+) Graph ~~rational functions~~, identifying zeros and asymptotes ~~when suitable factorizations are available~~, and showing end behavior.

e. Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.

Trigonometric Functions — F-TF

Model periodic phenomena with trigonometric functions

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

F-TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

Building Functions — F-BF

Build new functions from existing functions

F-BF.4 Find inverse functions.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In algebra 1, students analyzed linear, exponential, quadratic, absolute value, step, and piece-defined functions using different representations, and they interpreted linear, exponential, and quadratic functions that arose in applications within a context. Additionally, students learned, as general principle, the concept of a function and the use of function notation. In algebra 2, students analyzed and interpreted the functions previously studied with an emphasis on using key features to select appropriate models. Earlier in this course, students graphed functions identifying zeros, asymptotes, and showing end behavior (F.IF 4, 5, 6, 7a, 7b, 7c, 7d, 7e, 8a, 8b, 9).

In algebra 2, students extended the domain of trigonometric functions using the unit circle, modeled periodic phenomena with the sine and cosine functions, and proved and applied trigonometric identities. (F.T.F. 1, 2, 5)

In algebra 1, students built linear, exponential, and quadratic functions that model relationships between two quantities, and they built new functions using the basic operations from existing function types. Algebra 2 students built simple radical, rational, and exponential functions that model relationships, and they built new functions using the basic operations from existing function types studied. Earlier in this course, students determined inverses of functions, verified the inverse of a function using composition of functions, and produced invertible functions (F.BF 1a, 1b, 1c, 2, 3, 4a, 4b, 4c, 4d).

Current Learning

Students graph the six trigonometric functions by hand, identifying zeros, asymptotes, end behavior, period, midline, and amplitude. They model periodic phenomena determined by specified amplitude, frequency, and midline. Students identify inverse trigonometric functions through the graph of a trigonometric function and appropriately restrict the domain of the trigonometric function in order to determine the inverse of the trigonometric function.

Future Learning

Later in this course, students will apply these concepts when solving contextual problems involving trigonometric equations and using polar form.

Students will calculate values of trigonometric functions to evaluate definite integrals in calculus.

Additional Findings

According to *Principles and Standards for School Mathematics*,

- "The teacher could tell them that the length of daylight can indeed be modeled … parameters in physical equations have units" (p. 299).
- "High school students should be able to create and interpret models of more complex phenomena, drawn from a wider range of contexts, by identifying essential features of a situation and by finding representations that capture mathematical relationships among those features. They should recognize, for example, that phenomena with periodic features often are best modeled by trigonometric functions and that population growth tends to be exponential or logistic" (p. 361).

Precalculus, Quarter 2, Unit 2.3

Analytic Trigonometry

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Establish new identities based on previously learned trigonometric identities.
- Use sum and difference formulas to solve equations.
- Rewrite trigonometric expressions using trigonometric identities.
- Solve trigonometric equations using trigonometric identities.
- Solve trigonometric equations using inverse trigonometric functions.
- Solve applied problems using inverse trigonometric functions.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Build a logical progression of statements to explore the truth of identities.
- Justify conclusions, communicate them to others, and respond to arguments of others.
- Distinguish correct logic or reasoning from that which is flawed and explain why it is flawed.
- Determine the domains on which solutions apply for inverse trigonometric functions.

Model with mathematics.

- Use inverse functions to solve trigonometric equations that arise in modeling contexts.

Attend to precision.

- Use proper trigonometric vocabulary in discussion and in personal reasoning.
- Examine the claim that a trigonometric equation is an identity.

Essential questions

- How would trigonometric identities be used to simplify a more complex function?
- What real-world scenarios can be modeled using a trigonometric function?

Written Curriculum

Common Core State Standards for Mathematical Content

Trigonometric Functions	F-TF

Prove and apply trigonometric identities

F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Model periodic phenomena with trigonometric functions

F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In algebra 2, students extended the domain of trigonometric functions using the unit circle, modeled periodic phenomena with the sine and cosine functions, and proved and applied trigonometric identities. (F.T.F. 1, 2, 5, 8)

In algebra 1, students built linear, exponential, and quadratic functions that model relationships between two quantities, and they built new functions using the basic operations from existing function types. Algebra 2 students built simple radical, rational, and exponential functions that model relationships, and they built new functions using the basic operations from existing function types studied. Earlier in this course, students determined inverses of functions, verified the inverse of a function using composition of functions, and produced invertible functions (F.BF 1a, 1b, 1c, 2, 3, 4a, 4b, 4c, 4d).

Current Learning

Students prove trigonometric identities involving any of the six trigonometric functions. They use trigonometric identities to simplify expressions and solve equations. Students use inverse trigonometric functions to solve equations and problems. They are able to determine solutions using technology and interpret those solutions in the context of the problem.

Future Learning

Students will use trigonometric identities if they decide to pursue college level calculus.

Additional Findings

According to *Principles and Standards for School Mathematics*, “Reasoning and proof need to be a consistent part of student’s mathematical experience in prekindergarten through grade 12” (p. 56).

The book also states, “Systematic reasoning is a defining feature of mathematics” (p. 57).

According to *A Research Companion to Principles and Standards for School Mathematics*, “the functions of proof that may have the most promise for mathematics education are those of explanation and communication” (p. 228).

“The notion of proof is viewed as a central construct in mathematical thinking, and learning to understand and develop formal proofs is seen as an important aspect of student’s mathematical learning” (p. 228).

Teacher Notes:

Other possible content to include:

- Derive and use double-angle formulas to solve equations.
- Use half-angle formulas to solve equations.