Algebra 1, Quarter 2, Unit 2.1

Creating, Solving, and Graphing Systems of Linear Equations and Linear Inequalities

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Solve systems of linear equations graphically, including those that have infinitely many solutions and those that have no solution.
- Solve systems of linear equations using substitution and elimination and decide which of these methods is most appropriate.
- Write a system of linear equations and/or inequalities to fit a given situation.
- Solve systems of linear inequalities graphically and understand that the solution to the system is the region where the graphs of the individual inequalities overlap.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain which method of solving a system is most efficient, show work, and explain each step of the solution.
- Write systems of linear equations and inequalities that fit a given situation.

Model with mathematics.

- Choose the best method for solving linear systems which model a variety of situations, such as mixtures, rates, and break-even points.
- Use tables and graphs to visually represent algebraic solutions.

Attend to precision.

- Clearly state the meaning of symbols such as "greater than" and "less than or equal to" and temporarily replace inequalities with an equal sign during the graphing process.
- Use terms such as consistent, inconsistent, dependent, and independent to define and deepen the understanding of systems.

Look for and express regularity in repeated reasoning.

- When using the elimination method, look for and use least common multiples, add like terms, solve one-step equations, substitute, and check.
- When using the substitution method, solve for one variable, substitute into the other equation, solve multiple-step equations, substitute, and check.

Essential questions

- How would you describe the solution of a system of linear equations?
- How would you describe the solution of a system of linear inequalities?
- What does the solution of a system of linear equations mean?
- What does the solution of a system of linear inequalities mean?
- How do you know how many solutions a system will have?
- How do you determine the best method to solve a system of linear equations?
- How does “infinitely many solutions” differ from “all real numbers”?
- How does the solution of a system of linear equations compare and contrast to the solution of a system of linear inequalities?
- How can you use a system of linear equations or inequalities to model a real-world situation?

Written Curriculum

Common Core State Standards for Mathematical Content

Creating Equations★	**A-CED**

Create equations that describe numbers or relationships [*Linear~~, quadratic, and exponential~~ (integer inputs only); for A.CED.3 linear only*]

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.★

Reasoning with Equations and Inequalities	**A-REI**

Solve systems of equations [*Linear-linear ~~and linear-quadratic~~*]

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically [*Linear ~~and exponential~~; learn as general principle*]

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); ~~find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.~~ Include cases where f(x) and/or g(x) are linear, ~~polynomial, rational, absolute value, exponential, and logarithmic functions~~.

A-REI.12 ~~Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and~~ graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half –planes.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 8, students used systems of linear equations to represent, analyze, and solve a variety of problems. (8.EE.8c) They understood that a solution to a system of two linear equations in two variables corresponds to a point of intersection of a graph. (8.EE.8a) They also solved simple systems of two linear equations in two variables algebraically and by estimating solutions by graphing the equations. (8.EE.8b)

In the previous unit of this course, students graphed linear equations and inequalities in two variables. They graphed a single linear inequality in two variables in the coordinate plane by graphing its boundary line and then shading the half-plane. (A-REI.12)

Current Learning

Students determine the solutions to more complex systems of linear equations by graphing, substitution, and elimination. Students identify which method is most appropriate for a given system or a given situation. Students extend their knowledge of systems by solving systems of linear inequalities. Embedded in the unit, students create systems of linear equations and inequalities to model various situations.

Future Learning

Students will use methods of solving systems of equations and inequalities in algebra 2 when studying linear programming (unit 1.1). Solving systems is a skill that students will continue to use in subsequent course work as they solve systems of non-linear equations and inequalities.

Additional Findings

Principles and Standards for School Mathematics notes that in high school, students should build on their prior knowledge, learning more varied and more sophisticated problem-solving techniques. In addition, improving fluency with algebraic symbolism helps students represent and solve problems in many areas of the curriculum. Students should be able to operate fluently on algebraic expressions, combining them and re-expressing them in alternative forms (p. 288).

Algebra 1, Quarter 2, Unit 2.2

Interpreting Functions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand the concept of a function as assigning to each element of the domain exactly one element of the range.
- Use function notation and interpret statements that use function notation, including notation in terms of a given situation.
- Determine the domain and range of a given function (both algebraically and graphically) and a function in a given context.
- Evaluate functions for any input.
- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- Interpret key features of graphs and tables of functions (i.e., intercepts, intervals where the function is increasing, decreasing, positive or negative, relative maximums and minimums, and symmetries).
- Given key features of the functional relationship, sketch graphs.
- Calculate and interpret the average rate of change of a function over a specified interval.
- Estimate the rate of change from a graph.

Mathematical practices to be integrated

Attend to precision.

- Use clear definitions to explain reasoning when determining whether a relation is a function.
- Understand the key features of a function by correctly using related vocabulary.
- When determining rate of change of a function, specify appropriate units of measure.

Reason abstractly and quantitatively.

- Correlate the relationship among graphs, tables, and verbal descriptions of data.
- Explain what the changes in the graph of a function indicate about the rate of change of the function within different intervals of the domain.
- Look for patterns in sequences and represent them recursively.

Essential questions

- How do you determine the domain and the range of a function and what situations require you to look for restrictions on the domain and the range?
- How can you determine if a relation is a function?
- How can you describe a function using its key features?
- (*Given a verbal description*) How would you sketch a graph of this function?

- What is the advantage of using the notation $f(x) =$ as opposed to $y =$?
- How would you represent a sequence using function notation?
- How is the graph of a function affected if the rate of change of the function increases or decreases?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions	**F-IF**

Understand the concept of a function and use function notation [*Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences*]

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context [*Linear, exponential, and quadratic*]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries~~; end behavior; and periodicity~~.*★

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 8, students understood that a function is a rule that assigns to each input exactly one output. (8.F.1) They compared properties of two functions, such as the rate of change of each function when the two functions were represented in different ways. (8.F.2) By analyzing a graph, students described where a function was increasing or decreasing and if the function was linear or non-linear. (8.F.5)

Current Learning

Students determine if a relation is a function and determine the domain and range of a function. They use function notation to evaluate functions for inputs in the domain. They interpret statements that use function notation in terms of a context. They recognize that sequences are functions that can sometimes be defined recursively, whose domain is a subset of the integers. Students interpret key features of graphs and tables, and sketch graphs showing these key features. Students calculate and interpret the average rate of change of a function, and estimate the rate of change from a graph.

Future Learning

Students will continue to use and build upon their knowledge of functions throughout algebra 1, algebra 2, and precalculus. They will continue to graph by hand and to use technology for more complicated cases. Students will analyze functions and build functions to model relationships between two quantities. They will explore linear, exponential, quadratic, absolute value, step, piecewise-defined, logarithmic, rational, and trigonometric functions.

Additional Findings

Principles and Standards for School Mathematics notes that high school students should be able to interpret functions in a variety of formats. "Students should solve problems in which they use tables, graphs, words and symbolic expressions to represent and examine functions and patterns of change. Students should learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationships and as a style of mathematical thinking for formalizing patterns, functions, and generalizations" (p. 287).

By the completion of algebra 1. high school students should have substantial experience in exploring the properties of different classes of functions.

High school students' algebra experience should enable them to create and use tabular, symbolic, graphical, and verbal representations and to analyze and understand patterns, relations, and functions with more sophistication than in the middle grades.

Algebra 1, Quarter 2, Unit 2.3

Building and Graphing Functions

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Graph linear and quadratic functions showing intercepts, maxima, and minima.
- Graph exponential functions showing intercepts and end behavior.
- Graph absolute value and step functions.
- Utilize technology to graph complicated functions.
- Utilize technology to determine where the graphs of two functions intersect.
- Determine an explicit expression, a recursive process, or the steps for calculations from a given situation to write a function that describes a relationship between two quantities.
- Build a function to model situations by combining standard function types using arithmetic operations.
- Write arithmetic and geometric sequences with an explicit formula.
- Use arithmetic and geometric sequences to model situations.
- Identify the effect on the graph of replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$.
- Determine if a function is even or odd when the function is given graphically and algebraically.

Mathematical practices to be integrated

Model with mathematics.

- Graph functions expressed symbolically and show key features of the graph by hand or using technology.
- Graph exponential functions showing intercepts and end behavior.
- Graph linear and quadratic functions showing intercepts, maxima, and minima.
- Write a function that shows a relationship between two quantities.

Use appropriate tools.

- Use technology to graph more complicated functions.
- Use technology to experiment with cases and illustrate the effects on a graph of changing a function.
- Use technology to find points of intersection of two or more functions.

Essential questions

- How do you graph various types of functions (linear, exponential, quadratic, absolute value, step, piece-wise defined), and what are the key features of the graphs of these functions?
- How is a function affected by various parameter changes?
- What is the difference between a recursive formula and an explicit formula?
- When is it appropriate to use technology to graph functions?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions F-IF

Analyze functions using different representations [*Linear, exponential, quadratic, absolute value, step, piecewise-defined*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- e. Graph exponential ~~and logarithmic functions~~, showing intercepts and end behavior~~, and trigonometric functions, showing period, midline, and amplitude~~.
- b. Graph ~~square root, cube root, and piecewise-defined functions, including~~ step functions and absolute value functions.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). ~~*For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.~~

Building Functions F-BF

Build a function that models a relationship between two quantities [*For F.BF.1, 2, linear, exponential, and quadratic*]

F-BF.1 Write a function that describes a relationship between two quantities.★

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Build new functions from existing functions [*Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only*]

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Reasoning with Equations and Inequalities	**A-REI**

Represent and solve equations and inequalities graphically [*Linear and exponential; learn as general principle*]

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, ~~rational,~~ absolute value, exponential~~, and logarithmic~~ functions.★

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grade 8, students used functions to model relationships between quantities and constructed a function to model a linear relationship between two quantities. They determined the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. They described qualitatively the functional relationship between two quantities by analyzing a graph. (8.F.4 and 8.F.5)

In the previous unit of this course, students were introduced to function notation. They evaluated and interpreted functions, and determined the domain and range of a function.

Current Learning

Students graph linear and quadratic functions and show intercepts, maxima, and minima. Students graph exponential functions, absolute value functions, step functions, and piecewise-defined functions. Technology is used to graph more complicated cases and to also determine where the graphs of two functions intersect.

Students determine an explicit expression, a recursive process or the steps for calculation from a given situation in order to write a function that describes a relationship between two quantities. They build functions to model situations by combining standard function types using arithmetic operations. Students write arithmetic and geometric sequences both recursively and with an explicit formula. They use arithmetic and geometric sequences to model situations. Students identify the effect on the graph of replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$. Students determine if a function is even or odd.

Future Learning

In algebra 2, students will graph and identify key features of the parent functions $f(x) = x^2$, $f(x) = |x|$, $f(x) = \sqrt{x}$, and $f(x) = x^3$. In addition, students will continue to explore transformations of selected functions. They will also graph more advanced piecewise and step functions. (F.IF.4, F.IF.5, F.IF.7)

Additional Findings

Principles and Standards for School Mathematics notes:

"High school students' algebra experience should enable them to create and use tabular, symbolic, graphical, and verbal representations and to analyze and understand patterns, relations, and functions with more sophistication than in the middle grades" (p. 297).

"High school students should have substantial experience in exploring the properties of different classes of functions. For instance, they should learn that the function $f(x) = x^2 - 2x - 3$ is quadratic, that its graph is a parabola, and that the graph opens "up" because the leading coefficient is positive. They should also learn that some quadratic equations do not have real roots and that this characteristic corresponds to the fact that their graphs do not cross the x-axis. Students should also be able to identify the complex roots of such quadratics" (p. 299).