Precalculus, Quarter 3, Unit 3.1

Laws of Sines and Cosines and Area Formula

Overview

Number of instructional days: 8 (1 day = 45 minutes)

Content to be learned

- Derive and use the formula $y = \frac{1}{2}ab\sin(c)$ for the area of a triangle.
- Prove the Law of Sines and Law of Cosines.
- Apply the Law of Sines and Law of Cosines to solve nonright triangles.
- Use Heron's Formula to determine the area of a triangle. (Not in the CCSS)

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the meaning of a problem and determine which formula is needed to solve the problem.
- Routinely interpret mathematical results in the context of the situation and check reasonableness of solution.
- Consider the information given about the triangle to determine if an ambiguous case is present.

Construct viable arguments and critique the reasoning of others.

- Build a logical argument to derive the formula $y = \frac{1}{2}ab\sin(c)$.
- Prove the Law of Sines and Law of Cosines.
- Build a logical argument to determine which method will provide appropriate solutions.

Model with mathematics.

- Apply the Law of Sines and Law of Cosines to find unknown measurements of nonright triangles (e.g., surveying problems).
- Apply the appropriate area formulas (e.g., area of a parcel of land).

Attend to precision.

- Determine when to round and the appropriate number of decimal places necessary, depending on the context of the problem.

Essential questions

- What comparisons can be made between the formulas $y=\frac{1}{2}ab$ and $y=\frac{1}{2}ab\sin(c)$?
- Under what conditions is it necessary to use the Law of Sines? Law of Cosines?
- Under what conditions would it be necessary to use Heron's Formula as opposed to the formula $y=\frac{1}{2}ab\sin(c)$ for determining the area of a triangle?
- What real-world examples could be modeled using the Law of Sines or Law of Cosines?
- Under what conditions would solving a triangle be complicated by the ambiguous case?

Written Curriculum

Common Core State Standards for Mathematical Content

Similarity, Right Triangles, and Trigonometry	G-SRT

Apply trigonometry to general triangles

G-SRT.9 (+) Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems~~, resultant forces~~).

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 6, students determined the area of triangles. (6.G.1) In Geometry, Algebra 2, and previously in this course, students used trigonometric ratios and the Pythagorean Theorem to solve right triangles and applied problems. (G.SRT.8)

Current Learning

Students derive and use the formula $y = \frac{1}{2}ab\sin(c)$ for the area of a triangle. They create proofs for the Law of Sines and Law of Cosines. Students apply the Law of Sines and Law of Cosines to determine unknown measurements in right (see teacher notes) and nonright triangles. They use Heron's Formula to determine the area of a triangle. (Not in the CCSS)

Future Learning

Later in the vector unit in this course and in Calculus, students will use the Law of Sines and Law of Cosines in determining resultant vectors.

Additional Findings

"We use the Law of Cosines and Law of Sines to solve triangles that are not right-angled. Such triangles are called oblique triangles. The Law of Cosines is used much more widely than the Law of Sines …" (www.themathpage.com/atrig/law-of-cosines.htm)

Teacher Notes

- Standard G-SRT.11 states that the Law of Sines and Law of Cosines are taught for both right and nonright triangles. You can illustrate how the laws can be used in right triangles, but it results in a trivial case.
- Even though it is not in the standard, Heron's Formula should be instructed at this time.

Precalculus, Quarter 3, Unit 3.2

Polar Coordinate System

Overview

Number of instructional days: 12 (1 day = 45 minutes)

Content to be learned

- Convert rectangular coordinates and equations to the polar coordinate system.
- Represent and graph complex numbers in both the rectangular and polar coordinate system.
- Explain why the rectangular and polar forms of a given complex number represent the same number.
- Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane.
- Add, subtract, and multiply complex numbers using the polar coordinate system.
- Determine the roots and powers of complex numbers using DeMoivre's Theorem.
- Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Represent and graph complex numbers in both the rectangular and polar coordinate system.
- Determine the roots and powers of complex numbers using DeMoivre's Theorem.
- Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Model with mathematics.

- Represent and graph complex numbers in both the rectangular and polar coordinate system.
- Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane.

Essential questions

- Under what conditions is it recommended to change between rectangular and polar coordinates?
- Under what conditions is it more beneficial to use the polar coordinate system versus the rectangular coordinate system?

Written Curriculum

Common Core State Standards for Mathematical Content

The Complex Number System	N-CN

Represent complex numbers and their operations on the complex plane.

N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument* 120°.

N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In Algebra 2, students performed basic arithmetic operations (+, –, *) with complex numbers; they used complex numbers in polynomial identities and in solving quadratic equations and learned basic trigonometric identities. Previously in this course, students found the conjugates of complex numbers and used them to solve polynomial equations with real coefficients that have complex solutions. Given both real and complex roots of the equation, they also found polynomial equations. (N.CN.1, 2, 3, 7, 8)

Current Learning

Students convert rectangular coordinates and equations to the polar coordinate system and vice versa. They represent and graph complex numbers in both the rectangular and polar coordinate system. Students explain why the rectangular and polar forms of a given complex number represent the same number. They represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane. Students add, subtract, and multiply complex numbers using the polar coordinate system. They find roots and powers of complex numbers using DeMoivre's Theorem. They calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. (Students graph points and equations in the polar coordinate system. Not in standards.)

Future Learning

Students will use the concept of the polar coordinate system when studying vector space in this course and in physics. They will continue to use the polar coordinate system in calculus.

Additional Findings

"Applications: Polar coordinates are two-dimensional, and thus they can be used only where point positions lie on a single two-dimensional plane. They are most appropriate in any context where the phenomenon being considered is inherently tied to direction and length from a center point …" (http://en.wikipedia.org/wiki/Polar_coordinate_system#cite_note-21)

"For the operations of multiplication, division, and exponentiation of complex numbers, it is generally much simpler to work with complex numbers expressed in polar form rather than rectangular form." (http://en.wikipedia.org/wiki/Polar_coordinate_system)

Beyond Numeracy states, "The geometric representation of various operations in complex numbers paved the way for their indispensible use in electrical theory and other physical sciences." (p. 117)

Teacher Notes

This is a good opportunity to present graphs in the polar coordinate plane.

Precalculus, Quarter 3, Unit 3.3

Matrices

Overview

Number of instructional days: 10 (1 day = 45 minutes)

Content to be learned

- Multiply matrices by scalars to produce new matrices.
- Use matrices to represent and manipulate data.
- Add, subtract, and multiply matrices of appropriate dimensions.
- Calculate the value of the determinant.
- Determine the additive and multiplicative identity matrices.
- Determine the inverse of a matrix, when possible.
- Represent a system of linear equations as a matrix equation.
- Determine the solution to a 2×2 system of linear equations using inverse matrices.
- Use 2×2 matrices to transform a figure on the coordinate plane.
- Interpret the absolute value of the determinant in terms of area.
- Use technology to determine the solutions of 3×3 (or greater) systems of linear equations.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Determine whether a given matrix has an inverse.
- Interpret the solutions to a system of equations when the solution is not unique.

Use appropriate tools strategically.

- Represent a system of linear equations as a matrix equation.
- Determine the solution to a 2×2 system of linear equations using inverse matrices.
- Use 2×2 matrices to transform a figure on the coordinate plane.
- Use technology to determine the solutions of 3×3 (or greater) systems of linear equations.

Essential questions

- What process is used to determine the inverse of a matrix, when possible?
- What process would you use to determine the solution of a system of linear equations using matrices?
- Why does the Commutative Property not hold true for matrix multiplication?
- What are the similarities and differences between operations on matrices and operations on real numbers?

Written Curriculum

Common Core State Standards for Mathematical Content

Vector and Matrix Quantities	N-VM

Perform operations on matrices and use matrices in applications.

N-VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N-VM.12 (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Reasoning with Equations and Inequalities	A-REI

Solve systems of equations

A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grade 7, students were introduced to the concept of additive and multiplicative inverses. (7.NS.1.a, c) In Algebra 1, they solved systems of linear, linear-quadratic, and quadratic equations and inequalities. In Algebra 2, students solved systems of nonlinear equations and inequalities. (A.REI.5, 6, 7)

Current Learning

Students multiply matrices by scalars to produce new matrices, and they use matrices to represent and manipulate data. Students add, subtract, and multiply matrices of appropriate dimensions, and they calculate the value of the determinant. Students determine the additive and multiplicative identity matrices. They determine the inverse of a matrix, when possible. Students represent a system of linear equations as a matrix equation. They determine the solution to a 2×2 system of linear equations using inverse matrices. Students view 2×2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. They use technology to determine the solutions of 3×3 (or greater) systems of linear equations.

Future Learning

Students may use matrices in vectors and study of linear algebra.

Additional Findings

According to *Principles and Standards for School Mathematics*, “In grades 9–12 all students should understand vectors and matrices as systems that have some of the properties of the real number system.” (p. 393)

According to *Importance of Linear Algebra in Engineering Design Methodology* (Narayanan, Miami University [Ohio]), “Since the students who take precalculus have very little knowledge about the subject of matrices, it has become very important to treat the subject matter in depth. In 1989, the NCTM recognized the need for greater emphasis on linear algebra and stated that ‘matrices and their applications’ should receive ‘increased attention’ in high school curriculum. It should be recognized that linear algebra is as important as calculus to scientists and engineers.”

According to *Principles and Standards for School Mathematics*, “Students should also extend their understanding of operations to number systems that are new to them. They should learn to represent two-dimensional vectors in the coordinate plane and determine vector sums. Dynamic geometry software can be used to illustrate the properties of vector addition. As students learn to represent systems of equations using matrices, they should recognize how operations on the matrices correspond to manipulations of such systems.” (p. 293)

Teacher Notes

In an advanced class, students may need to solve 3×3 systems of linear equations using expansion by minors. This method will be used when instructing the cross product of three-dimensional vectors.

N-VM.12 (+) ***Work with 2 × 2 matrices as transformations of the plane***, and interpret the absolute value of the determinant in terms of area.

From N-VM.12, the bold text references performing transformation on the plane such as reflections, rotations, and translations.

Precalculus, Quarter 3, Unit 3.4

Representing and Modeling with Vector Quantities

Overview

Number of instructional days: 11 (1 day = 45 minutes)

Content to be learned

- Represent vector quantities by directional line segments.
- Use appropriate symbols for vectors and their magnitudes.
- Determine the components of a vector using the initial and terminal points.
- Add and subtract vectors.
- Multiply vectors by a scalar.
- Multiply a vector by a matrix of suitable dimensions.
- Solve problems involving velocity and other quantities that can be represented by vectors.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Solve problems involving velocity and other quantities that can be represented by vectors.

Model with mathematics.

- Represents force, velocity, surveying, navigation, and other examples with vectors.

Essential questions

- When would vectors be used in the real world?
- When adding two vectors together, why is the magnitude of the resultant vector not the same as adding the magnitudes of the original vectors?
- What characterizes a vector in a plane?
- Why will the magnitude of the resultant vector always be smaller than the sum of the magnitudes of two forces?

Written Curriculum

Common Core State Standards for Mathematical Content

Vector and Matrix Quantities	**N-VM**

Represent and model with vector quantities.

N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, */**v**/*, *//**v**//*, *v)*.

N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

N-VM.4 (+) Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction ***v*** – ***w*** as ***v*** + (–***w***), where –***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.5 (+) Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $||c\mathbf{v}|| = |c|v$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|v \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

N-VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Students were introduced to trigonometric functions in Algebra 2 and in this course. This material will be used when determining the magnitude and direction of vectors.

Current Learning

Students represent vector quantities by directional line segments, and they use appropriate symbols for vectors and their magnitudes. Students determine the components of a vector using the initial and terminal points. They add and subtract vectors and multiply vectors by a scalar. Students multiply a vector by a matrix of suitable dimensions. They solve problems involving velocity and other quantities that can be represented by vectors.

Future Learning

Students may use this concept if they take a physics or linear algebra course.

Additional Findings

According to *Principles and Standards for School Mathematics*, “In grades 9–12 all students should understand vectors and matrices as systems that have some of the properties of the real-number system.” (p. 393)

“Learning vector algebra represents an important step in students' ability to solve problems. The importance of vector algebra can be understood in the context of previous …” (www.nhn.ou.edu/walkup/demonstrations/WebTutorials/VectorIntroduction.htm)

According to *Principles and Standards for School Mathematics*, “Students should also extend their understanding of operations to number systems that are new to them. They should learn to represent two-dimensional vectors in the coordinate plane and determine vector sums. Dynamic geometry software can be used to illustrate the properties of vector addition. As students learn to represent systems of equations using matrices, they should recognize how operations on the matrices correspond to manipulations of such systems.” (p. 293)

According to *Knowledge Construction in High School Physics: A Study of Student Teacher Interaction* (Wessel), “The students did not understand the process of mathematical representation to any great extent, nor did they understand that direction is a fundamental characteristic of many physics concepts. These two factors combined to make the use of vector mathematics even more difficult for the participants and most grade 12 physics students. The participants lacked a perception of any need for vectors or vector mathematics. They did not have a sense of why they had been taught about vectors in other courses nor could they describe any practical applications when we talked about vectors early in the study. …

Most science teachers do not have the arsenal of instructional strategies and experience necessary to create these experiences for students because the type of instruction that I am advocating had not been used to any extent in science education. This result will also have ramifications for teacher education programs.”
(www.saskschoolboards.ca/old/ResearchAndDevelopment/ResearchReports/Instruction/99-04.htm#apply)

Teacher Notes

Dot products and cross products could be introduced at this point in the curriculum.