

# Precalculus, Quarter 4, Unit 4.1

## Probability and Statistics

### Overview

**Number of instructional days:** 14 (1 day = 45 minutes)

#### Content to be learned

- Apply the general Multiplication Rule in a uniform probability model.
- Use permutations and combinations to compare probabilities of compound events.
- Use probabilities to make fair decisions.
- Analyze decisions and strategies using probability concepts.
- Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space.
- Graph the corresponding probability distribution using the same graphical displays as for data distributions.
- Calculate the expected value of a random variable and interpret it as the mean of the probability distribution.
- Develop a probability distribution in which theoretical probabilities can be calculated or probabilities are assigned empirically.
- Weigh the possible outcomes of a decision by assigning probabilities to payoff values.
- Find the expected values of probability distributions.
- Evaluate and compare strategies on the basis of expected values.

#### Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Use conditions to determine which probability model is appropriate for a given situation.
- Make an argument for the analysis of a data set supported by conditions and probability models.

Model with mathematics.

- Construct probability models based on a data set.
- Apply the probability models to a given scenario and make predictions and generalities about the data set.

Use appropriate tools strategically.

- Use the appropriate probability model for a given situation.
- Use appropriate technology to visualize probability models.

**Essential questions**

- In what real-world context would it be possible to use probability concepts to make fair decisions?
- In what real-world context would it be possible to use probability concepts to analyze decisions and strategies?
- Under what conditions would it be possible to calculate theoretical probability?
- Under what conditions would it be necessary to have probabilities assigned empirically?
- In what real-world context would it be possible to use probability concepts to find the expected payoff for a game of chance?
- In what real-world context would it be possible to use probability concepts to evaluate and compare strategies on the basis of expected values?

## Written Curriculum

**Common Core State Standards for Mathematical Content****Conditional Probability and the Rules of Probability\*****S-CP**

**Use the rules of probability to compute probabilities of compound events in a uniform probability model**

- S-CP.8 (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.\*
- S-CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.\*

**Using Probability to Make Decisions\*****S-MD**

**Use probability to evaluate outcomes of decisions**

- S-MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).\*
- S-MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).\*

**Calculate expected values and use them to solve problems**

- S-MD.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.\*
- S-MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.\*
- S-MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing*

*on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.\**

- S-MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?\**

### Use probability to evaluate outcomes of decisions

- S-MD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.\*
- Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
  - Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

## Common Core Standards for Mathematical Practice

### 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of

the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

### **Clarifying the Standards**

#### *Prior Learning*

In grades 6 and 7, students computed measures of center and variation, made informal inferences about a population of interest, and developed an understanding of probabilities and probability models. In Geometry, they were introduced to the concepts and applications of conditional probability. In Algebra 2, students were introduced to the normal model and standard deviation.

#### *Current Learning*

Students apply the general Multiplication Rule in a uniform probability model. They use permutations and combinations to compare probabilities of compound events. Students use probabilities to make fair decisions, and they analyze decisions and strategies using probability concepts. Students define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space. They graph the corresponding probability distribution using the same graphical displays as for data distributions. Students calculate the expected value of a random variable and interpret it as the mean of the probability distribution. They develop a probability distribution in which theoretical probabilities can be calculated or probabilities are assigned empirically. Students weigh the possible outcomes of a decision by assigning probabilities to payoff values. Students find the expected values of probability distributions, and they evaluate and compare strategies on the basis of expected values.

#### *Future Learning*

Students may use this material in future statistics courses as a foundation for statistical inference.

### **Additional Findings**

According to *Navigating through Probability in Grades 9–12*, “Statements of probability appear frequently in newspapers and magazines. Educated readers should be able to understand how reported probabilities, such as the probability of being struck by lightning, of being involved in an automobile accident, or of winning the lottery, have been determined and can be interpreted. Students need to develop an understanding of basic concepts of probability so that they can reason successfully about uncertain events, assess risks, and grasp the logic of making decisions in the presence of uncertainty.” (p. 4)

According to *Principles and Standards for School Mathematics*, “In high school, students can apply the concepts of probability to predict the likelihood of an event by constructing probability distributions for simple sample spaces. Students should be able to describe sample spaces such as the set of possible outcomes when four coins are tossed and the set of probabilities for the sum of the values on the faces that are down when two tetrahedral dice are rolled.

High school students should learn to identify mutually exclusive, joint, and conditional events by drawing on their knowledge of combinations, permutations and counting to compute the probabilities associated with such events.” (p. 331)



## Precalculus, Quarter 4, Unit 4.2

# Sequences and Series

### Overview

**Number of instructional days:** 12 (1 day = 45 minutes)

#### Content to be learned

- Construct arithmetic, geometric, recursive, and other sequences.
- Calculate sums of finite and infinite series.
- Express series in sigma notation.
- Determine convergence and divergence.
- Prove theorems by mathematical induction.

#### Mathematical practices to be integrated

Look for and make use of structure.

- Determine whether a sequence is arithmetic or geometric.
- Develop a recursive and explicit definition for a sequence or series.
- Use the structure of proof by mathematical induction.

Look for and express regularity in repeated reasoning.

- Analyze sequences by differences or ratios.
- Determine convergence/divergence and intervals of convergence.

#### Essential questions

- What is the graphical representation of an arithmetic or geometric sequence?
- Why do the terms of a geometric sequence decrease in magnitude when  $-1 < r < 1$ ?
- Why must mathematical induction have an initial condition?

## Written Curriculum

### Common Core State Standards for Mathematical Content

Sequences—No CCSS Standard

Arithmetic Sequences—No CCSS Standard

Geometric Sequences—No CCSS Standard

Geometric Series—No CCSS Standard

### Common Core Standards for Mathematical Practice

#### 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

#### 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

### Clarifying the Standards

#### *Prior Learning*

In grade 3, students recognized patterns in arithmetic sequences. In grades 4 and 5, they generated and analyzed patterns in arithmetic and geometric sequences. In Algebra 1, students constructed arithmetic and geometric sequences.

*Current Learning*

Students construct arithmetic, geometric, recursive, and other sequences. Students calculate sums of finite and infinite series, and they express series in sigma notation. Students determine convergence and divergence, and they prove by mathematical induction.

*Future Learning*

Sequences and series are topics in second-level calculus to approximate polynomials and other functions. The limit of a series leads to integral calculus such as Riemann sums.

**Additional Findings**

“Series is one of those topics that many students don’t find all that useful. To be honest, many students will never see series outside of their calculus class. However, series do play an important role in the field of ordinary differential equations and without series large portions of the field of partial differential equations would not be possible.

In other words, series is an important topic even if you won’t ever see any of the applications. Most of the applications are beyond the scope of most Calculus courses and tend to occur in classes that many students don’t take. So, as you go through this material keep in mind that these do have applications even if we won’t really be covering many of them in this class.”

(<http://tutorial.math.lamar.edu/Classes/CalcII/SeriesIntro.aspx>)



## Precalculus, Quarter 4, Unit 4.3

# Introduction to Calculus

### Overview

**Number of instructional days:** 10 (1 day = 45 minutes)

#### Content to be learned

- Determine continuity of functions.
- Evaluate limits of functions as  $x$  approaches  $c$ .
- Evaluate the average rate of change of a function over a closed interval.
- Evaluate the instantaneous rate of change of a function at a point (as a limit).

#### Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Determine when to use average rate of change versus instantaneous rate of change.

Reason abstractly and quantitatively.

- Determine when to use average rate of change versus instantaneous rate of change.
- Interpret the limit of a function at a given point, if it exists.
- Decontextualize an application, manipulate the symbols, and use appropriate operations.

Model with mathematics.

- Illustrate the limit in a variety of ways such as graphically, algebraically, in table form, and using technology.

#### Essential questions

- How are the average rate of change and instantaneous rate of change similar? Different?
- Under what conditions could the graph of a function be discontinuous?
- What is a limit of a function?

## Written Curriculum

### Common Core State Standards for Mathematical Content

Average Rate of Change—No CCSS Standard

Limits—No CCSS Standard

Instantaneous Rate of Change—No CCSS Standard

### Common Core Standards for Mathematical Practice

#### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions

and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### Clarifying the Standards

#### *Prior Learning*

In grade 8, students learned about slope and linear functions. In Algebra 1, they explored the behavior of linear, quadratic, and exponential functions. In Algebra 2, students analyzed polynomial, rational, exponential, and logarithmic functions. Earlier in this course, different types of continuity were discussed.

#### *Current Learning*

Students determine continuity of functions. Students evaluate limits of functions as  $x$  approaches  $c$ , and they evaluate the average rate of change of a function over a closed interval. They evaluate the instantaneous rate of change of a function at a point (as a limit).

#### *Future Learning*

In Calculus, students will continue the study of limits and rate of change as the derivative and the integral.

### Additional Findings

“Reaction rates have the general form of (change of concentration/change of time). There are two types of reaction rates. One is called the average rate of reaction, often denoted by  $(\Delta[\text{conc.}]/\Delta t)$ , while the other is referred to as the instantaneous rate of reaction, denoted as either:

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta[\text{concentration}]}{\Delta t} \quad \text{or} \quad \frac{d[\text{concentration}]}{dt}$$

The average rate of reaction, as the name suggests, is an *average* rate, obtained by taking the change in concentration over a time period, for example:  $-0.3 \text{ M}/15 \text{ minutes}$ . This is an approximation of the reaction rate in the interval; it does not necessarily mean that the reaction has this specific rate throughout the time interval or even at any instant during that time. The instantaneous rate of reaction, on the other hand, depicts a more accurate value. The instantaneous rate of reaction is defined as the change in concentration of an infinitely small time interval, expressed as the limit or derivative expression above. Instantaneous rate can be obtained from the experimental data by first graphing the concentration of a system as function of time, and then finding the slope of the tangent line at a specific point which corresponds to a time of interest. Alternatively, experimenters can measure the change in concentration over a very small time period two or more times to get an average rate close to that of the instantaneous rate. The reaction rate for that time is determined from the slope of the tangent lines.”

([http://chemwiki.ucdavis.edu/Physical\\_Chemistry/Kinetics/Reaction\\_Rates/Measuring\\_Reaction\\_Rates#Average\\_and\\_Instantaneous\\_Reaction\\_Rate](http://chemwiki.ucdavis.edu/Physical_Chemistry/Kinetics/Reaction_Rates/Measuring_Reaction_Rates#Average_and_Instantaneous_Reaction_Rate))

